Use of Reinforced Epoxy Models to Design and Analyze Aircraft Structures

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Aluminum-reinforced epoxy models are being used as precursors to prototype design in the development and modification of aircraft structures. In conjunction with experimental strain-measurement methods, these models provide extensive design information before engineering, tooling, and manufacturing expenses are incurred. Test applications, model fabrication, and experimental stress-analysis methods are discussed to illustrate the potential uses of the technique. Material characteristics such as time-modulus criteria together with the effects of model rework and transducer reinforcement are developed to acquaint the potential user with technical restraints.

Introduction

E CONOMIC and competitive pressures repeatedly force the structures engineer to make quick and accurate decisions about structural integrity. Experimental stress-analysis techniques can be used to establish design criteria for the modification of existing designs and the analysis of new designs, to improve production reliability and reduce weight and costs. Photoelastic-coating and/or strain-gage tests are normally performed on a finished preproduction assembly, however, drawing releases, forgings, and many production units can be completed before a design limitation is discovered; and consequent design changes and scrappage can cause cost increases. Three-dimensional (3-D) photoelastic analysis has been widely used to combat this situation, but it is limited by the size and complexity of the model and by the number of loading conditions that can be investigated.

A designer often uses a layout prototype in wood, clay, or cardboard to obtain a three-dimensional visualization of his conceptual drawings. To prove the integrity of his design, he often needs another model for kinematic and structural verification, in a material that is elastic, homogeneous, and easily stress-analyzed. Solid metal models can satisfy the elasticity requirements, but the cost of machining, the cost of full-load test fixtures, the time delays involved in modifications, and the difficulty of adding material to metals have limited their use. A single plastic model can be used for both a kinematic study and structural verification, substantially reducing not only the cost of model making, but also the cost and time of testing, because a much lighter test fixture can be used. The major problem with a plastic model is creep under load.

To overcome this difficulty, the Airplane Division of The Boeing Company has developed a new concept in design and stress analysis employing an epoxy reinforced with 25–45% by weight of an aluminum powder. The homogeneous mix has a density of 0.055 lb/in.³ compared to 0.043 for a solid epoxy and 0.1 for aluminum. The Aluminum Reinforced

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Epoxy (ARE) has a modulus of elasticity E ranging from 0.35 to 1.30 \times 106 psi and an ultimate tensile strength σ_u ranging from 4000 to 8000 psi. New molding techniques were developed to reduce model costs and the time required to produce a finished ARE model. An important advantage of ARE models is the ability to add or remove material easily.

Typical Aircraft Applications

In 1961, the Boeing 707 main and nose landing gear were investigated by this approach, which is outlined in Fig. 1. In 1964, the VERTOL Division applied the technique to helicopter components, which are subjected to an infinite number of low-to-medium stress cycles rather than a finite number of high stress cycles. The landing gear of the Boeing 720, 727, 737, and 747 also were evaluated by ARE model analysis. Figure 2 shows the fullscale (20-ft-high) model that was used to perfect the landing gear for both the 710,000-lb 747 which just entered passenger service and the advanced 747B which will weigh 775,000 lb. More than 900 different model parts were required to complete the assembly of the 747 nose, body, and wing landing gear.

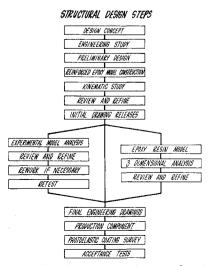


Fig. 1 Structural design steps, using an aluminum reinforced epoxy (ARE) model at an early stage.

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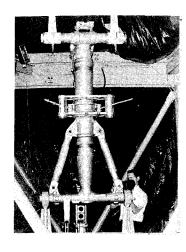


Fig. 2 ARE model of the Boeing 747 landing gear.

Field problems in Vietnam necessitated a major redesign of a helicopter's transmission cover. It was redesigned with a four-point mounting and subjected to a rigorous ARE model stress analysis, stages of which were undertaken while the transmission cover was being forged to the approximate finished dimension and a test rig was being designed and fabricated. Application of the scaled design loads to the model revealed stress concentrations on the right-aft and leftforward lug arms near the bolt circle. Material was removed from these areas, and the subsequent photoelastic survey of the model showed that: 1) peak stresses had been lowered by ~25% 2) the critical load path had been moved away from the bolt circle. The modified configuration, which utilized less material, was incorporated into the already released design drawings. Subsequent fatigue tests established that fatigue life was more than doubled.

The ARE model technique is occasionally combined with 3-D photoelastic analysis of an epoxy resin model and a photoelastic coating analysis of the finished assembly (Fig. 1). Figure 3 displays three swashplates: one completely cast from ARE, another cast from epoxy resin, and the final steel component with a photoelastic coating applied. (The swashplate is a spherically mounted bearing and control lever used to transmit control motions from the stationary control linkages within the helicopter fuselage to the moving rotary wing assembly.)

A relatively complex ARE model is shown in Fig. 4. The model of a hub assembly, representing a new concept in helicopter rotor blade attachment, was made for a kinematic study prior to the dynamics investigation of blade motions. It was made directly from sketches; all parts were manufactured first in wax and then molded in aluminum epoxy. Structures also can be built up using ARE bar and sheet stock. This technique is being developed in France to test components of a supersonic aircraft. Many similar components have been manufactured to simulate linkages and support structures found in helicopters and aircraft.

Material Properties

Properties and behavior of most structural materials are well documented, but this is not the case with model mate-

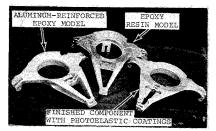


Fig. 3 Models and prototype of a helicopter swashplate.



Fig. 4 ARE model of a helicopter hub.

rials. Accurate design data were required on E, σ_u , compressive ultimate strength, shear modulus, and Poisson's ratio ν . Much of the ARE material experimentation was directed toward the development of a suitable and compatible E. In order for the model/design concept and translation to prototype stresses to be completely valid, it is necessary for the mechanical properties to be similar in both tension and compression. True similitude of stresses also requires compatible ν .

For each model the modulus-time relationship is determined by machining tensile coupons from the same batch of material as the part to be tested, bonding strain gages to them in a Poisson arrangement, applying dead load, and recording strains for one hour. After allowing sufficient time for recovery (~2 hr), the coupon is tested at a different load. Strain values at 1, 10, and 60 min are obtained from the strain-time curves and are used to plot stress-strain curves at these times. The apparent E's computed from slopes from these curves are used to prepare a correction curve for viscoelastic creep vs time, because ARE's do not conform to Hooke's Law throughout the entire elastic range. For loads from $\sim 0.1 \sigma_u$ to $\sim 0.5 \sigma_u$ (the safe test limit) the stress-strain curve is reasonably straight, however. Various ARE's with different resin/hardener combinations have been tested with the possibility of incorporating them into composite structures of different moduli. The hardener/resin ratios were also evaluated for homogeneity, castability, and over-all test application. If a material is not homogeneous, variations in E will cause test errors. To evaluate the homogeneity of a given cure of aluminum epoxy material, multiple vertical and horizontal specimens were cut from a block. Tests of these specimens showed coefficients of variation of 0.6% for E and 8.5% for σ_u . (Since all testing should be performed at stresses considerably below σ_u , this σ_u variation is unimportant.) Repeated tests of multiple specimens of a single mold verifies this low variation in E. However, greater variations will occur between mixes or batches, and E should be determined for each mold pour.

To determine ν , strain gages were applied to specimens to monitor both ϵ_1 and ϵ_2 by means of an X-Y plotter. A constant strain at different time intervals was applied. For both commonly used resin/hardener combinations, $0.33 \leq \nu \leq 0.35$.

Model Preparation and Application

For the initial models, solid blocks of ARE were vacuumcured and then machined. The material is easy to machine, nonbrittle, and does not possess the low melting point which is characteristically detrimental to machining thermoplastics. But machining was expensive and time-consuming, so plaster molds, formed by using an aircraft prototype modified with wax, or a separate wooden or wax model as in the case of a completely new design configuration, was tried. However, plaster molds are usually destroyed during removal of the finished model, and they have a tendency to leak during the casting process. These problems were overcome by developing split-mold, silastic envelope techniques, which also can be used to make low cost, 3-D photoelastic models. Sheet and bar stock can be used to model panels and simple structures by bonding to simulate welded joints and using machined or molded ARE screws to simulate bolts or rivets.

Viscoelastic Creep Correction

Viscoelastic creep can produce an error of as much as 15% in strain measurement if the proper precautions are not taken. The creep-time characteristics depend on the exact ARE formulation used, how it is cured, and the temperature during testing.

Figure 5 is a typical time-creep correction curve. modulus at 1 min (from the application of full load) is $1.15 \times$ 106 psi and is used as the base value. The continuing high creep rate between 1 min and 10 min is indicated by the fact that the correction factor has changed from 1.0 to 0.9. exact work can be done simply by recording time simultaneously with each strain gage or photoelastic reading, and then using the creep correction curve for the material being tested. Thus, the strain can be immediately corrected at the same time the conversion is made to prototype stresses. For general laboratory testing, a simpler procedure suffices, because for the time span 15-45 min, only a 2% change occurs; thus, E can be expressed in terms of its 30-min figure, and the testing for each model be conducted between 15 and 45 min after load application. The material is allowed to relax for a period twice that of the application and test time before another load condition is applied. This procedure eliminates continuous creep correction.

When many loading conditions must be investigated in a short time, it may be necessary to apply the loads in successively increasing increments with no relaxation periods. By utilizing the laws of superposition, creep correction factors were obtained for successively applied loads by the relation

$$C_f = [P_0(t_i - t_0)^{-n} + P_1(t_i - t_j)^{-n} + P_2(t_1 - t_2)^{-n} + \dots P_j(t_i - t_j)^{-n}]/\Sigma_0 P_j \quad (1)$$

where t_j is the time of application of each additional increment of load P_j . Preliminary application of Eq. (1) has corresponded with the standard creep correction method for uniform load increments of from 15- to 60-min duration.

Addition or Removal of Material, and Bonding

To obtain stress continuity between a filler and the model, the fill material must exhibit modulus and creep characteristics similar to those of the model material. The best results have been obtained using a formulation close to that of the parent material. In filling thinner wall sections a small error (5%) may result, which in most cases is less than experimental error. Exact results can be obtained by another model after the optimum configuration is found, if this accuracy is deemed necessary.

Rework can be accomplished by removing the photoelastic coating and/or strain gage and forming beeswax to the model until the desired configuration is attained. A mold can be fabricated on this modified area. After curing, the mold is released, and the clay or wax can then be removed and weighed to determine the equivalent weight increase to the actual component as a result of the rework. The mold is then reassembled and filled to produce the new configuration. Filler material also can be applied in bulk form and then filed, machined, or sanded to shape. In general, material can be added to ARE models with justification in small areas where estimates of stress distribution are desirable.

Material can be removed easily by plastic working hand tools or machines. No adverse effects result, and an rms 30 or better finish may be obtained by sanding or filing. No additional surface preparation is necessary prior to the application of strain gages or photoelastic coatings to this reworked surface.

Extensive tests have shown that a butt joint in a test area will not appreciably affect test results (3% max error). If the bonding is not in a test area, the process will not affect the results and the structure will be as strong in the bonded area as in the parent material.



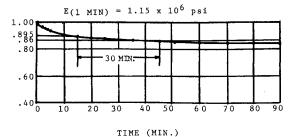


Fig. 5 Creep correction factor vs time.

Homogeneity and Dimensional Control

Since the epoxy material as poured is viscous, poor molding techniques can lead to voids. In most cases, the voids appear as surface pinholes and can be filled or sanded away. A second, less probable, homogeneity error is the presence of unpolymerized material. All models are examined by x-rays prior to assembly and stress analysis. Voids and regions of unpolymerized material as small as $\frac{1}{16}$ in. across are revealed by x-rays at depths of 3 in. or more, whereas defects that have been corrected by refilling are almost undetectable. Specimens should be x-rayed before and after any bonding or filling operation.

Shrinkage of the base resin is 0.0004 in./in. Dimensional problems are primarily inherent in the accuracy of the mold material. The model should be built up in any close-tolerance areas and then machined. However, with normal care, mold models can be held to $\pm \frac{1}{16}$ in. with no special precautions. Finishing operations can be done easily using standard machine-shop equipment and simple hand tools, although best results are achieved with diamond or tungsten carbide tips. Finish cuts are usually held to 0.005–0.030 in., depending on the rigidity of the model and cutter. The speed should be medium with low rates of feed.

Model Test Fixture

Since $0.35 \times 10^6 \le E \le 1.3 \times 10^6$ psi for ARE's, model-tocomponent strain similitude can be obtained by the applying to the model of a force approximately $\frac{1}{20}$ that required for the metal component. Therefore, the test fixture usually can be designed and assembled quickly and at low cost, using commercially available predrilled erector-set-type angles, channels, and square stock, together with air cylinders and pressure regulators to apply loads. The only machined piece usually needed is the model attachment fitting for the load application and reaction. If possible, the model material should be extended past the point of interest to prevent any fixture restraint due to modulus difference. Testing should incorporate a complete ARE system where feasible; e.g., in a rotor blade attachment structure, ARE bushings, bearings, pins, and bolts were utilized to simulate local bearing stresses and deflections. When simulating composite structures or different moduli, the ARE formulation should be varied to obtain the desired modulus ratios.

Photoelastic Coatings and Strain Gages

When a plastic, birefringent coating is applied to an ARE model, the coating will sustain a portion of the applied stresses and thus modify the original stress system. The effect is particularly significant for thin sections and an ARE model having a modulus comparable to that of the coating. Also, the coating/ARE combination represents a nonisotropic structure that is difficult to analyze directly. This problem is handled by reducing the composite structure to an equivalent one having isotropic properties. Figure 6 shows the reinforcement factors vs coating/model thickness ratio for plane

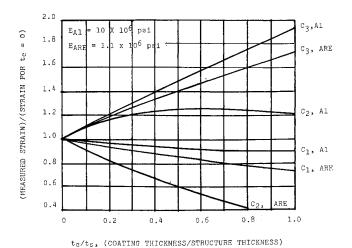


Fig. 6 Coating correction factors: C_1 for plane stress, C_2 for flexure under constant load, C_3 for constant-deflection bending.

stress, flexure under a constant load and constant-deflection bending.

The reinforcing effect can be minimized by using the thinnest coatings commensurate with reasonable fringe orders. The most desirable photoelastic operating range is that which produces two fringes. Therefore, where a relatively thick model is being investigated, a coating is selected to produce this birefringence. However, for a thin model, especially one subjected to bending, it is preferable to use a thinner coating. This may result in some loss of sensitivity, but operation in the high correction area of the bending correction curve is avoided. If more accurate results are necessary, the coating should be replaced by strain gages in the areas of stress concentration.

The application of strain gages is often desirable to automate data collection and simplify separation of principal stresses. The analysis of strain-gage data on ARE models, while not nearly as difficult as the interpretation and analysis of strain-gage data on solid plastic models, can still be a source of error. The thermal conductivity of an ARE is about 1/100 that of steel; hence, heating of the strain gage due to excessive currents can cause localized heating of the ARE and the following problems: change in apparent strain, change in strain-gage output, and change in ARE properties. To minimize this effect, the bridge supply voltage should be kept as low as possible, consistent with adequate signal readout.

Time and Cost Comparisons

The component used for this evaluation of alternative experimental analysis techniques is a helicopter transmission cover, a magnesium forging that is machined to its final configuration. For each of the five test plans considered, it is assumed that a first model of some sort (plaster, wood, etc.) is necessary for a fit and function and/or kinematic analysis, and that final acceptance testing (in this case fatigue testing) of the

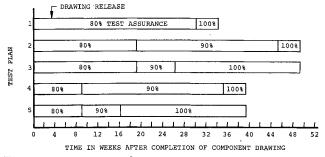


Fig. 7 Comparison of five test plans: elapsed time vs assurance of design adequacy.

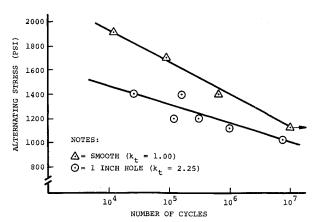


Fig. 8 S-N curves for ARE coupons without and with a hole.

production component is necessary. The additional models used in the five test plans are as follows: 1) none; 2) aluminum machined model; 3) Al machined model and 3-D photoelastic model; 4) ARE model; 5) ARE model and 3-D photoelastic model. Both machined-aluminum and molded ARE use photoelastic coating and strain-gage analysis. Figure 7 illustrates on a time bar chart the percent assurance of acceptability the designer would have at various times after release of the drawings. The completion of a 3-D model to check internal stress normally increases assurance from 80% to 90%. A 3-D analysis by itself will not produce a high assurance of adequacy, since each model can be subjected to only one load condition and an over-all view of the stress distribution and concentrations is not possible. The completion of all testing represents a 100% test assurance of success. The time required to obtain 80% assurance of success is reduced from 30 weeks when no experimental analysis is performed, to 19 weeks when an aluminum model is used, to 9 weeks with an ARE model. The importance of this time saving is apparent when a forging production schedule is reviewed. In most cases no metal has been forged at the 9-week point. Changes at this stage are relatively cheap and uncumbersome. Often, changes can be made to effect cost and weight savings which would not normally be feasible under another type of test plan.

Note that the shortest over-all schedule is that which encompasses no testing. The additional time and cost required to conduct an experimental analysis must be considered as insurance to obtain an adequate, economical, and safe design. When amortized over the first lot of 100 forgings, the cost of machining an aluminum specimen, procuring a test fixture, and conducting an experimental stress analysis represents a manufacturing cost increase of $\sim 5\%$; the ARE model, only 2.5%. The cost of design, stress analysis, etc. has not been included in this analysis. The authors believe that the increasing possibility of a poor design caused by the advancing state-of-the-art, and the time involved to conduct a thorough theoretical stress analysis, amply justify an experimental model analysis, and that the time and cost involved represent cheap insurance. In the Boeing Company today, ~50% of high-fatigue-loaded components are subjected to a preproduction experimental stress analysis. In several years, this number probably will increase to 90%.

Epoxy Models for Fatigue Analysis

An ARE model can be statically tested to the different scaled loads, and the steady and alternating stresses at any location can be calculated from this analysis. If the load inputs are such that a vectorial analysis cannot be made of the resultant strains for the different load inputs, the model can be tested by applying alternating loads to obtain dynamic strain magnitudes and directions at critical areas. Creep and

calibration constants must be adjusted in proportion to the rate of test loading. A test engineering challenge for the future is the use of an ARE model to obtain a fatigue evaluation prior to the completed component fatigue test. For example, if one needs to determine fatigue strengths of various complex hole geometries subjected to a biaxial complex load pattern, and if these geometries can be machined or molded from an ARE, it is possible that fatigue results can be obtained with ARE. Preliminary test investigations in this area, using ARE bars in a smooth and a hole configuration, have yielded promising results (Fig. 8), but many problems such as notch sensitivity, aging, brittleness, and model similitude yet require detailed investigation.

Conclusions

This ARE model technique constitutes an important laboratory tool. Development of an ARE with an average 15-min modulus of over 1 \times 106 psi and an average ultimate strength of 7000 psi has provided the necessary structural material. An ARE model provides not only true perspective visualization for the designer after he has completed preliminary drawings, but also a fast, inexpensive means of assessing structural integrity and for adding or removing materials to proportioned, the resultant stress distribution will be identical to those of its metal counterpart. These molded models are easy to machine, easy to rework, and simple to load.

The primary disadvantage of the material is inherent in its viscoelastic properties. Further studies of the complex characteristics and variations in materials and mixes, and further development on conversion of model results to accurate final-configuration strains, are needed. However, for a general understanding of stress distribution and concentrations, and a quick comparison of configurations and geometry,

the process is considered developed.

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Stress Concentration Factors for Bonded Lap Joints

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This paper is concerned with the stress analysis by the finite element method of a bonded, single, lap joint. Since the adhesive layer is of primary importance, the stresses occurring in it are presented. A modified version of the well-known Wilson stress analysis program is used for the case of plane stress. The total length beyond the lap is considered long compared to the lap length. Stress concentrations as functions of dimensionless, geometric and material parameters are presented. For a given load σ , the important maximum shearing stress concentration, $au_{
m max}/\sigma$, and tearing stress concentration $\sigma_{
m max}/\sigma$, are plotted as functions of l/t and η/t for different values of E/E_A , where l is the lap length, η is the adhesive thickness, t is the thickness of the material being bonded, and E and E_A are the moduli of elasticity for the adherend and the adhesive, respectively.

Nomenclature

= applied stress = shear stress

= tearing stress

 $\tau_{\rm max} = {\rm maximum \ shear \ stress}$ = maximum tearing stress

= length of each end section

= length of lap

= adherend thickness

= adhesive thickness

E= adherend elastic modulus

 E_A = adhesive elastic modulus

Poisson's ratio

Introduction

F considerable importance in the design of a bonded joint is the decision as to which type of joint would be most advantageous. Of the more common types, the lap joint and its variations are presently the most practical and most studied. The lap joint can be basically one of two types, the single lap joint, with one bondline, and the double lap joint, with two bondlines.

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The double lap joint in tension is the simpler type to analyze since there is no bending incurred during deformation, and considerable work has been done in this area. 1,4,10

Of more practical importance is the single lap joint, which has likewise received considerable attention. The first reliable treatment of the problem was given by Goland and Reissner⁵ who performed an analytical stress analysis of cemented lap joints in 1944. However, the simplifications necessary to make the analytical solution possible have restricted the results. Basically, Goland and Reissner considered two extreme cases. One was based on the assumption that most of the deformation occurred in the adhesive layer, as in metal to metal bonding; the other treated an inflexible adhesive layer, as in a wood to wood joint. A few years later Cornell³ studied a simplification of Goland and Reissner's problem by replacing the adhesive with a system of tension and shear springs. Thus, his analysis neglected the Poisson's ratio effect, and the stresses parallel to the joint. Subsequently, many papers have been published, all with analytical or experimental simplifications. 2,6-9 Little work has been done in this area using numerical techniques, and to the



Fig. 1 Physical problems.